Nonequilibrium Field Theory of Open Quantum Fluids

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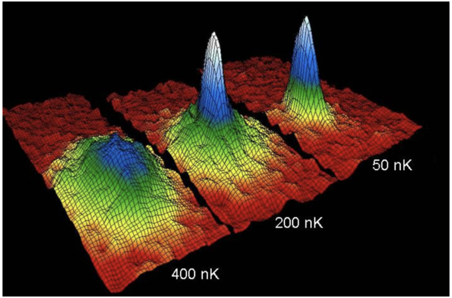
Atomic condensate as an open quantum system



$$i\hbar\partial_t\psi = \left[\underbrace{\left(-\frac{\hbar^2\nabla^2}{2m} + V_{\text{trap}}\right)}_{\text{single-particle } H_{\text{sp}}} + g|\psi|^2\right]\psi.$$

closed, coherent field evolution

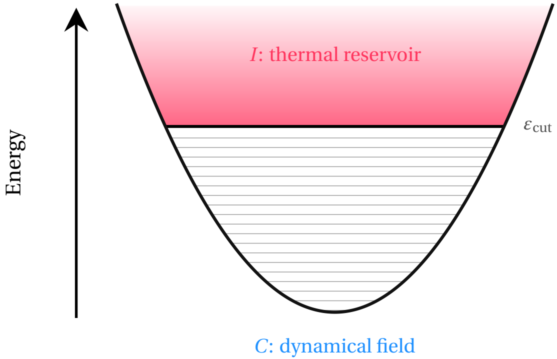
Thermal matter waves

- condensate formation
- dissipative dynamics

Zero-T Gross–Pitaevskii

- fixed atom number
- no reservoir damping or noise

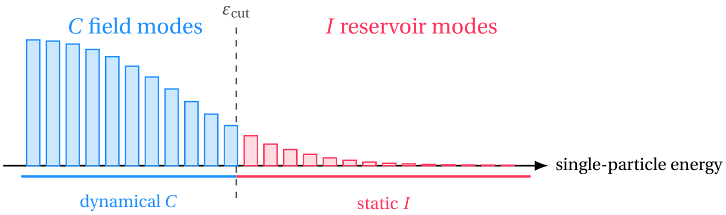
Energy separation



- equilibrium at high energy
- dynamics at low energy
- the cutoff $\varepsilon_{\mathrm{cut}}$ is formally carried through the theory
- quantitative, first-principles reservoir theory

C. W. Gardiner and M. J. Davis, *J. Phys. B* **36**, 4731 (2003).

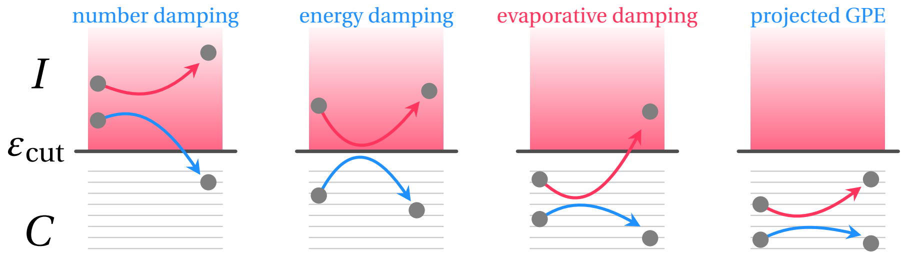
Projection decides what is dynamical



$$\psi(\mathbf{r},t) = \sum_{\varepsilon_n<\varepsilon_{\rm cut}} \alpha_n(t)\phi_n(\mathbf{r}), \qquad \mathscr{P}^2 = \mathscr{P}.$$

- The cutoff sets the dynamical C-field.
- The location of $\varepsilon_{\rm cut}$ is based on mode population (truncated Wigner).
- The static I region has parameters T, μ, setting damping and noise.

The reservoir has three channels



Number damping (γ)

- Particles cross between C and I. Equivalent to imaginary time GPE.

Energy damping (ε)

- Thermal scattering damps motion in C.

Evaporative damping (γ, ε)

- A nonlinear collision sends one atom across the cutoff.

C. W. Gardiner and M. J. Davis, *J. Phys. B* **36**, 4731 (2003).
A. S. Bradley, M. J. Davis, and C. W. Gardiner, *Phys. Rev. A* **77**, 033616 (2008).
N. A. Krause and A. S. Bradley, *Phys. Rev. A* **113**, 043311 (2026).

The SPGPE for all channels

$$(S)\,d\psi = \mathscr{P}\Bigg\{\underbrace{-\frac{i}{\hbar}(L-\mu)\psi\,dt}_{\text{projected GPE}} + \underbrace{\frac{\gamma}{k_BT}(\mu-L)\psi\,dt + dW_\gamma}_{\text{number damping}}$$

$$+\underbrace{-\frac{i}{\hbar}V_\varepsilon\psi\,dt + i\psi\,dW_\varepsilon}_{\text{energy damping}}\Bigg\}, \qquad L = H_{\mathrm{sp}} + g|\psi|^2.$$

Closed part

- Hamiltonian flow with no thermal sampling.

Dissipative part

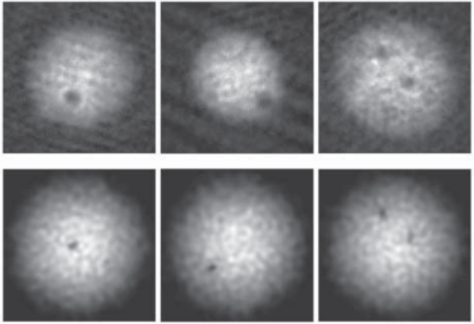
- Each drift channel has paired noise.

Projector

- Every term is returned to the C region.

Spontaneous vortices as BEC forms

experimental observations

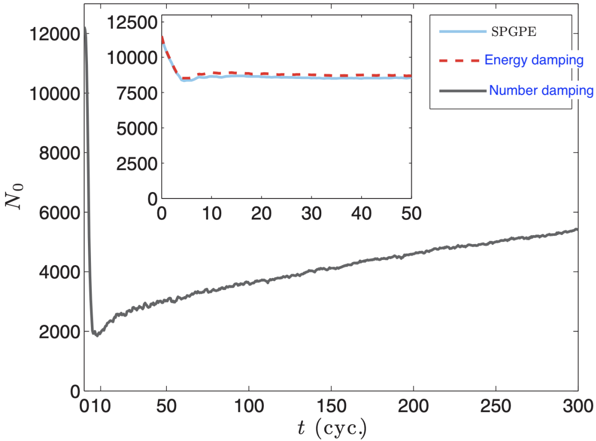


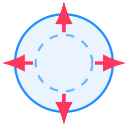
SPGPE simulations

- A finite quench through T_c outruns critical relaxation; domains freeze in with length $\hat{\xi}$ (Kibble–Zurek mechanism).
- Random phases merge; closed phase loops become vortices with $n_v \sim \hat{\xi}^{-2}$.
- SPGPE remains valid through T_c: it evolves the noisy field as growth and defects emerge.

C. N. Weiler, T. W. Neely, D. R. Scherer, A. S. Bradley, M. J. Davis, and B. P. Anderson, *Nature* **455**, 948 (2008).

Breathing separates mechanisms





radial breathing changes density

- Number-damping dynamics relaxes on the wrong time scale.
- Energy damping removes collective energy; it damps the current carried by the breathing mode.
- Energy damping is coherent: little condensate loss, unlike number damping.

S. J. Rooney, P. B. Blakie, and A. S. Bradley, *Phys. Rev. A* **86**, 053634 (2012).

Energy damping acts through the current

$$\boxed{(S)\,d\psi\big|_{\varepsilon} = \mathscr{P}\left\{-\frac{i}{\hbar}V_{\varepsilon}\psi\,dt + i\psi\,dW_{\varepsilon}\right\}}$$

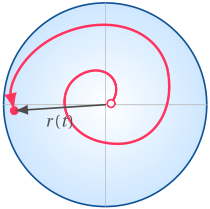
$$V_{\varepsilon}(\mathbf{r},t) = -\hbar\int d^3\mathbf{r}'\,\varepsilon(\mathbf{r}-\mathbf{r}')\,\nabla'\cdot\mathbf{j}(\mathbf{r}',t).$$

$$\mathbf{j} = \frac{\hbar}{2mi}\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right) = n\mathbf{v}.$$

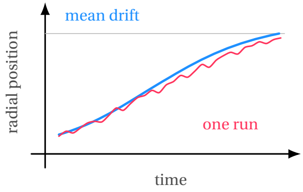
- The scattering potential is sourced by current divergence.
- Static density alone does not trigger energy damping.
- The same kernel sets the noise associated with the energy-damping mechanism, dW_{ε}.

A. S. Bradley, C. W. Gardiner, and M. J. Davis, *Phys. Rev. A* **77**, 033616 (2008).

Mutual friction makes vortices spiral out

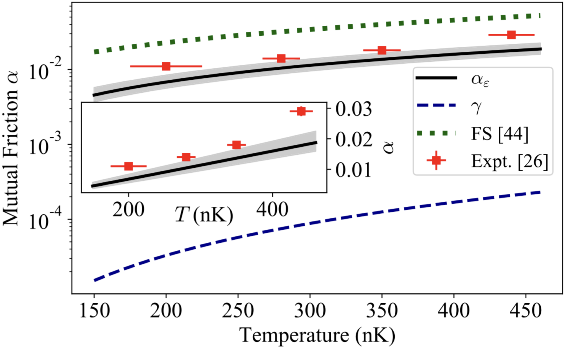


damped vortex spirals to the edge



- A vortex in a trapped condensate precesses around the density gradient.
- Thermal friction deflects that motion into radial drift.
- Observable is radial position versus time; fluctuations give diffusion around the mean drift.

Mutual friction from energy damping



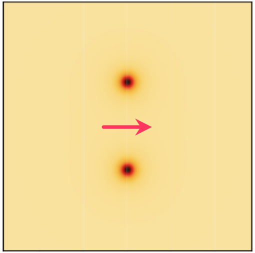
- Thermal drag on vortices is mutual friction; the SPGPE gives an effective point-vortex model.

$$d\mathbf{r}_n = [\mathbf{v}_n^0 - \alpha_\varepsilon q_n \hat{\mathbf{z}} \times \mathbf{v}_n^0]\, dt + \sqrt{2\eta}\, d\mathbf{W}_n.$$

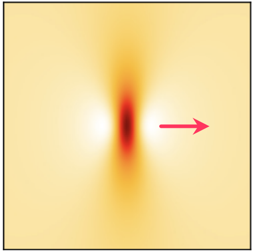
- mutual friction derived in closed form from V_ε
- Noise is fixed by the friction.

Z. Mehdi, J. J. Hope, S. S. Szigeti, and A. S. Bradley, *Phys. Rev. Research* **5**, 013184 (2023).
G. Moon, W. J. Kwon, H. Lee, and Y. Shin, *Phys. Rev. A* **92**, 051601(R) (2015).

Jones–Roberts solitons



vortex dipole

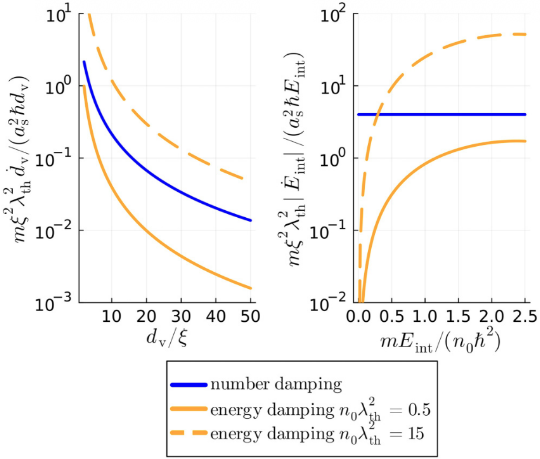


rarefaction pulse

- Classs of excitations ranging from vortex dipoles to rarefaction pulses.
- Spanning from incompressible to compressible excitations.
- Energy damping acts through density-changing motion.

N. A. Krause and A. S. Bradley, *Phys. Rev. A* **110**, 053302 (2024).

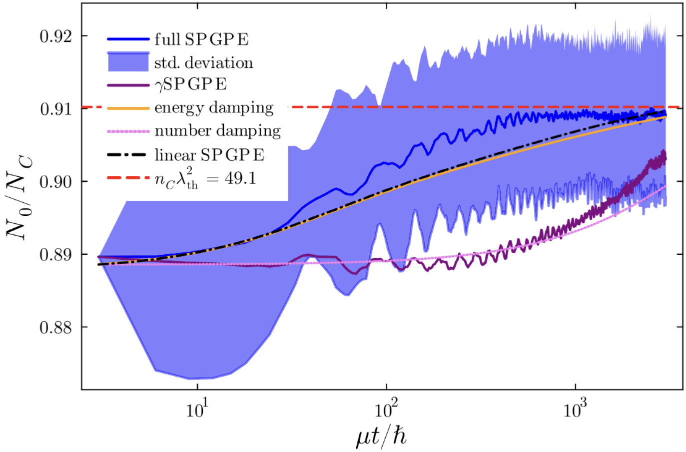
Phase-space density sets damping mechanism



- Phase-space density $n_0\lambda_{th}^2$ controls mechanism.
- Energy damping is the dominant mechanism for high phase space density.
- Holds in both dipole and rarefaction pulse regimes.

N. A. Krause and A. S. Bradley, *Phys. Rev. A* **110**, 053302 (2024).

Temperature quench in a homogeneous gas

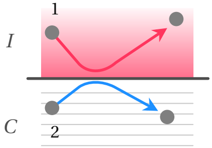


- Full SPGPE growth is faster than number damping alone.
- The linear theory predicts the near-equilibrium trend.
- Energy damping speeds up thermalisation by an order of magnitude.

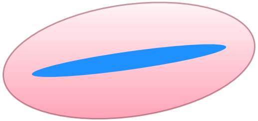
N. A. Krause and A. S. Bradley, *Phys. Rev. A* **112**, 033308 (2025).

Components, dimensions, solitons, cooling

Two-component system



Low-dimensional SPGPE

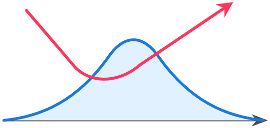


1D field in a 3D reservoir

Sympathetic cooling mechanism

- The target species interacts with a distinct thermal species.
- The only reservoir interaction is of energy-damping type.

Bright-soliton damping



bright soliton density

A. S. Bradley and P. B. Blakie, *Phys. Rev. A* **90**, 023631 (2014).
A. S. Bradley, S. J. Rooney, and R. G. McDonald, *Phys. Rev. A* **92**, 033631 (2015).
R. G. McDonald and A. S. Bradley, *Phys. Rev. A* **93**, 063604 (2016).

Outlook and summary

Summary

- Projection defines the C field; the I region fixes T, μ, damping, and noise.
- **Number damping** grows the field through T_c; **energy damping** relaxes it after BEC forms.
- Energy damping dissipates currents, vortices, and collective modes without the **decoherence** of atom loss.

Outlook

- Quantitative tests of damping mechanisms in experiments are needed.
- Test order-of-magnitude thermalisation speedups in homogeneous gases.
- **Reduced stochastic models** for defects and coherent structures can be derived analytically for **energy damping**.

References

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